Total and Merchantable Volume Equations for Small Coastal Douglas-fir

by
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December 1987

ECONOMIC & REGIONAL DEVELOPMENT AGREEMENT

Canada

BC

Partial funding for this research project and the cost of printing this publication were provided by the Canada/British Columbia Forest Resource Development Agreement - a five year (1985-90) $300 million program cost-shared equally by the federal and provincial governments.

Canadian Cataloguing in Publication Data

Omule, Stephen Agnew Yen'Emurwon, 1952-
Total and merchantable volume equations for small Coastal Douglas-fir

(FRDA report, ISSN 0835-0752 ; 010)

Co-published by B.C. Ministry of Forests and Lands.
Issued under Forest Resource Development Agreement.
On cover: Canada/B.C. Economic & Regional Development Agreement.
Bibliography: p.
ISBN 0-7718-8598-9

1. Douglas-fir - British Columbia - Measurement. I. Fletcher, Val Edward, 1946- . II. Polsson, K. R. III. Canadian Forestry Service. IV. British Columbia. Ministry of Forests and Lands. V. Forest Resource Development Agreement (Canada) VI. Canada/BC Economic & Regional Development Agreement. VII. Title. VIII. Series.

SD397.D7048 1987 634.9'754285'09711 C87-092163-0

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Province of British Columbia

This is a joint publication of the Canadian Forestry Service and the British Columbia Ministry of Forests and Lands.

Produced and distributed by the Ministry of Forests and Lands, Research Branch.

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ABSTRACT

Based on a sample of 2563 trees accumulated from a variety of sources in coastal British Columbia, the prediction bias of the existing total and merchantable volume equations (BCFS 1976; Demaerschalk & Kozak 1977; Kovats 1977; Godfrey, unpubl. memo, 1983; pers. comm., 1983) were evaluated for small coastal Douglas-fir. The equation of BCFS (1976) was generally unbiased for total volume prediction, for trees with diameter 5 cm or more; however, the equations of Demaerschalk and Kozak (1977) and of Kovats (1977) consistently under-predicted total volume, and that of Godfrey (pers. comm. 1983) over-predicted total volume. All the equations were biased for trees with diameter below 5 cm. Demaerschalk and Kozak (1977) under-predicted the merchantable volume of smaller trees and over-predicted that of larger trees. Godfrey (unpubl. memo, 1983) consistently over-predicted merchantable volume for all trees. New total and merchantable volume prediction equations were developed for trees less than 45 cm in diameter. It is recommended that existing volume equations be evaluated for trees larger than 45 cm in diameter for coastal Douglas-fir, as well as for the other significant species in the province.

Keywords: Tree volume equations; merchantable volume; coastal Douglas-fir.

ACKNOWLEDGEMENTS

We gratefully acknowledge the co-operation of the following individuals during collation of the data used in this study: G. Godfrey (Research Branch consultant), C. Ying, M. Kovats, and K. Richardson (Inventory Branch). We also thank J.P. Demaerschalk (University of British Columbia), G. Godfrey, K. Iles (MacMillan Bloedel Ltd.), G.R. Bailey (Valuation Branch), G.M. Bonnor (Pacific Forestry Centre), and M. Kovats for reviewing an earlier draft of this paper; G. Montgomery for doing the editorial review; and E. Lomas for word processing.

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INTRODUCTION

Several published tree volume equations are currently used in British Columbia, some in summaries of appraisal and inventory cruise data and some in studies of growth and yield and tree improvement of coastal Douglas-fir. These include: British Columbia Forest Service (BCFS) (1976), Demaerschalk and Kozak (1977), Kovats (1977), and Godfrey[1,2] (Table 1). The BCFS (1976) total volume equations are based on a logarithmic model, and apply to a wide array of forest inventory zones and age classes. They were developed from a sample originating largely from natural stands and containing a high proportion of large trees. Merchantable volume factors, which can be applied directly to total volume to give merchantable volume, are also tabulated in BCFS (1976). However, the methodology for derivation of these tables was not documented. Demaerschalk and Kozak (1977) developed a system for predicting tree profiles based on the same data as used by BCFS (1976). It consists of two mathematical functions, one for the upper bole and the other for the lower bole of a tree, which are linked and continuous at the inflection point. Total and merchantable volumes are obtained by integrating the two functions. Demaerschalk and Kozak's (1977) system is apparently not suitable for very small trees (e.g., less than 7.5 m tall or 10 cm in diameter). Kovats (1977) developed an equation, also based on a logarithmic model, using a variable reference height for diameter measurement, for trees under 10 m tall. He based this on about 100 trees taken mainly from plantations. Godfrey (pers. comm., 1983), using an expanded data set which consisted of a large number of small trees from thinning studies, as well as Kovats' (1977) and BCFS (1976) data, developed a total volume logarithmic model. In addition, he fitted a mathematical function to the tabulated merchantable volume factors (BCFS) (1976), making interpolation of these tables easier and more precise (Godfrey, unpubl. memo, 1983).

The BCFS (1976), Demaerschalk and Kozak (1977), and Godfrey (pers. comm., 1983) volume prediction equations are generally unsuitable for estimating volumes of small trees (e.g. under 5 cm in diameter) because of limitations of the model or sample data used. Kovats' (1977) equation, although specifically developed for small trees, was based on a relatively small sample.

A rare opportunity to evaluate the existing tree volume prediction equations for coastal Douglas-fir was provided by a new large data set (about 1400 trees) recently acquired by B.C. Ministry of Forests and Lands, Research Branch, in addition to the data used by the previous analysts. The objectives of the study were to evaluate the prediction bias of the existing total and merchantable volume equations for coastal Douglas-fir small trees, arbitrarily defined here as trees having a diameter at breast height (dbh) of less than 45 cm. (The 45 cm diameter limit represented the maximum diameter of the bulk of the newly acquired data.) In addition, new volume prediction equations were developed for the small trees, based on the expanded data set, to be used in summaries of Research Branch growth and yield plot data.

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[1] Godfrey G., personal communication, 1983, B.C. Ministry of Forests and Lands, Research Branch, Victoria, B.C.

[2] Godfrey, G. 1983. Merchantable volume factors (MVF) for immature trees. Smoothing out the steps. B.C. Ministry of Forests and Lands, Research Branch, Victoria, B.C. Unpublished memo, dated 1983-10-27, File EP 705.

TABLE 1. Coastal Douglas-fir tree volume equations

Source	Equation	Standard error of estimate	No. of trees in sample
BCFS(1976)	$\log_{10}(V) = -4.319071 + 1.813820 \log_{10}(D) + 1.042420 \log_{10}(H)$ where V = total volume inside bark in cubic metres D = diameter breast height outside bark in centimetres H = total height in metres	10.3%	393
Demaerschalk and Kozak (1977)	Computerized taper model — volumes obtained by integration		393
Kovats (1977)	$\ln(V) = -3.33875 + 1.70435 \ln(d) + 1.38551 \ln(H) + 2.30499 (h/H)$ where V = total volume inside bark in cubic centimetres d = diameter outside bark in centimetres at height h h = distance in centimetres from point of germination to the point of diameter (d) measurement H = total height in centimetres	10.29%	100
Godfrey[a]	$\ln(V) = -9.872837 + 1.869321 \ln(D) + 0.967596 \ln(H)$ where V, D, and H are as defined in BCFS (1976)	0.0877 m^3	630
Godfrey[b]	$MV = V [0.9563(1-e^{-0.3770(D-9.004)0.660})]$ for $9.1 \leq D \leq 32.4$ $= 0$ for $D \leq 9.0$ cm (For $D \geq 32.5$, use BCFS tabulated values) where MV = merchantable volume in cubic metres defined in BCFS (1976)		-

[a] Godfrey G., personal communication, 1983, B.C. Ministry of Forests and Lands, Research Branch, Victoria, B.C.

[b] Godfrey, G. 1983. Merchantable volume factors (MVF) for immature trees. Smoothing out the steps. B.C. Ministry of Forests and Lands, Research Branch, Victoria, B.C. Unpublished memo, dated 1983-10-27, File EP 703.

SAMPLE DATA

The study data consist of 2563 trees accumulated from several sources in coastal British Columbia (Table 2). Data were screened to ensure that trees having a dbh of 45 cm or more were excluded. Sample data were plotted on a relative scale to show similarities in shape between different tree size classes (Figure 1). All data were depicted graphically on a height - diameter - volume chart (Figure 2). Imperial measurements, including those of breast height (4.5 ft), were converted to metric before analysis.

TABLE 2. Sources and characteristics of study data

Data Source	Number of trees	Number of sections per tree	dbh (cm)			Total ht (m)		
			Min.	Mean	Max.	Min.	Mean	Max.
BCFS Inventory Branch volume & decay study trees: Forest Inventory Zones A,B,C (immature/mature age classes)	476(520)[a]	8-12	5.4	26.7	44.8	6.6	23.2	44.0
Douglas-fir provenance trial (EP[b] 599:Port Renfrew)	1313(0)	13	3.9	14.6	23.9	3.1	11.2	16.0
Douglas-fir inbreeding program (EP 474:Cowichan Lake)	51(0)	11-15	17.4	21.3	30.5	12.0	15.0	17.6
Tree Improvement program (EP 685:Cowichan Lake)	54(0)	12	1.5	2.6	4.6	2.0	2.6	3.5
Volume study — plantation & natural trees (variable sites)	129(0)	8-18	0.5	6.2	16.4	1.4	5.1	12.2
Sayward Forest (EP 678)	115(4)	11	6.7	19.0	44.2	8.7	17.2	32.6
MacMillan Bloedel sample plot near Powell River	67(4)	6-11	17.7	23.9	41.3	23.8	15.2	32.2
Forest Productivity Committee thinning study (EP 703) (variable sites)	257(0)	10-18	5.0	14.8	43.3	4.3	14.4	26.9
Thinning study (EP 418:Cowichan Lake)	27(0)	13	14.7	26.9	42.2	15.4	24.3	29.0
Thinning study (EP 554:Sayward Forest)	74(0)	10	11.5	20.0	30.4	12.9	18.2	24.5
All data	2563	8-18	0.5	17.1	44.8	1.4	14.1	44.0

[a] The values in brackets are numbers of trees with dbh 45 cm or more.

[b] EP = B.C. Ministry of Forest and Lands Research Branch Experimental Project.

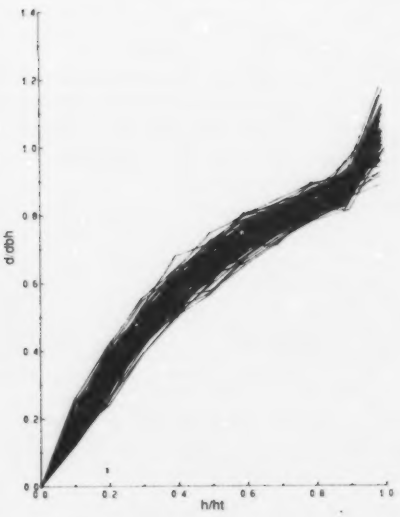

FIGURE 1. Coastal Douglas-fir stem profiles on a relative scale. EP554.

d = any upper stem diameter, outside bark at height h
dbh = diameter at breast height outside bark
ht = total height
h = height from tree tip to point of diameter d measurement

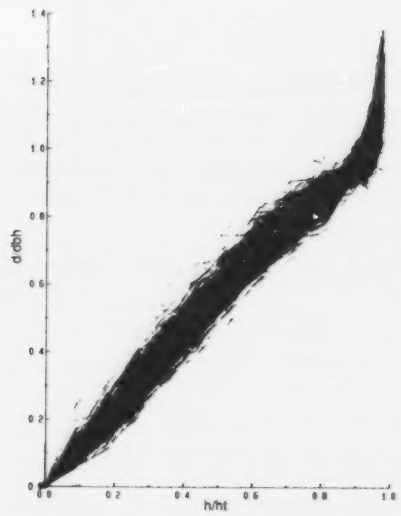

FIGURE 1. (Cont'd) Coastal Douglas-fir stem profiles on a relative scale. EP 599.

d = any upper stem diameter, outside bark at height h
dbh = diameter at breast height outside bark
ht = total height
h = height from tree tip to point of diameter d measurement

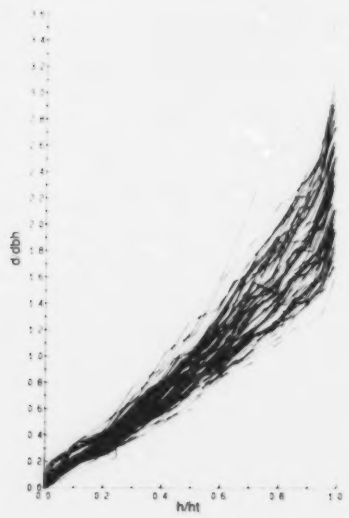

FIGURE 1. (Concl.) Coastal Douglas-fir stem profiles on a relative scale.
EP685.

d = any upper stem diameter, outside bark at height h
dbh = diameter at breast height outside bark
ht = total height
h = height from tree tip to point of diameter d measurement

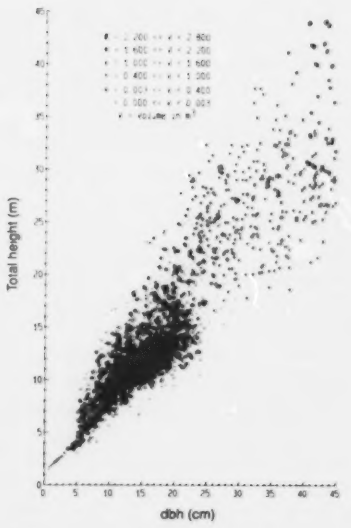

FIGURE 2. Height - diameter - volume frequency distribution.

ANALYSIS AND RESULTS

Evaluating Existing Volume Equations

Total volume

Prediction bias was evaluated by comparing "true" volume to "estimated" volume. The true total volume inside bark was calculated for each tree by summing the sectional volumes which, for all but the stump and top sections, were calculated according to Smalian's formula. The top and stump section volumes were calculated using the formula for volume of a cone and cylinder, respectively. The estimated total volume was obtained from each of four systems of prediction equations (BCFS 1976; Demaerschalk and Kozak 1977; Kovats 1977; Godfrey, pers. comm., 1983) based on tree dbh and total height.

Two measures of bias were used. Actual bias was calculated as tree volume minus estimated volume, and relative bias was calculated as actual bias divided by true volume. These measures were plotted against dbh, separately and for each prediction equation, for visual inspection. Significance of actual bias was tested by fitting a simple linear regression of estimated volume on true volume, and using the t-test to detect whether the slope of the regression was statistically different from unity (p=0.05). If it was not, the regression intercept was tested to see if it was statistically different from zero. Regression slope and intercept of unity and zero, respectively, define a line of correspondence between the true and estimated volumes.

The volume equation of BCFS (1976) showed no consistent patterns of bias for trees of dbh 5 cm or more (Figure 3), while Godfrey's (pers. comm., 1983) and Demaerschalk and Kozak's (1977) prediction equations, respectively, consistently over-predicted and under-predicted total volume in this diameter range (Figures 4 and 5). However, for trees below 5 cm in diameter, all three prediction systems were biased. Kovats' (1977) equation consistently under-predicted total volume (Figure 6). These observations from the graphical displays of relative biases were supported by results of the t-tests (Appendix 1), which showed the bias to be statistically significant for total volume estimation based on the Kovats (1977) and Demaerschalk and Kozak (1977) equations, and not significant for Godfrey (pers. comm., 1983) equation. (Graphical displays of actual biases are not shown.)

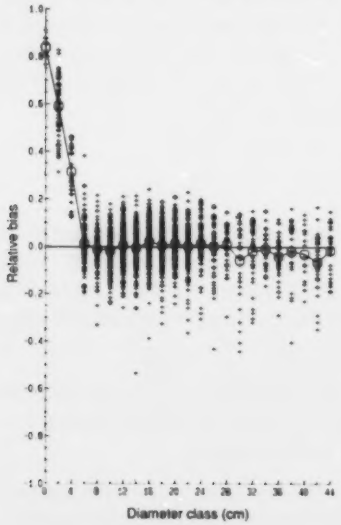

FIGURE 3. Relative bias by diameter classes of BCFS (1976) total volume equation.
Relative bias = (True volume) - (Estimated volume)/True volume.
0 = diameter-class mean relative bias.

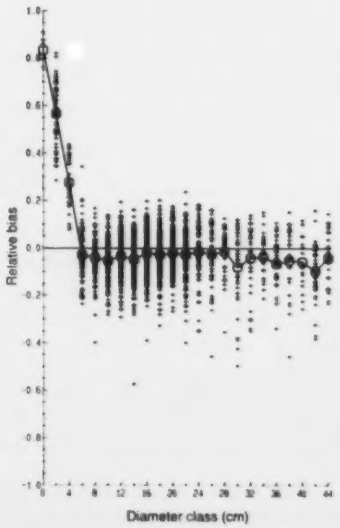

FIGURE 4. Relative bias by diameter classes of Godfrey (pers. comm., 1983) total volume equation.
Relative bias = (True volume) - (Estimated volume)/True volume.
0 = diameter-class mean relative bias.

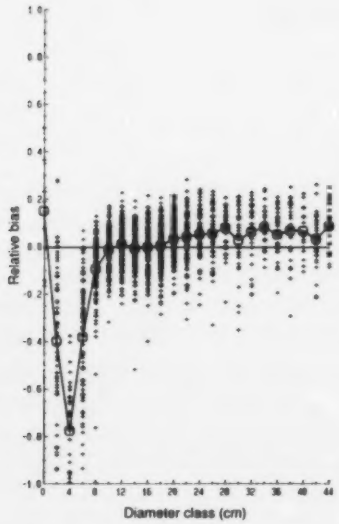

FIGURE 5. Relative bias by diameter classes of Demaerschalk and Kozak (1977) total volume prediction system.
Relative bias = (True volume) - (Estimated volume)/True volume.
0 = diameter-class mean relative bias.

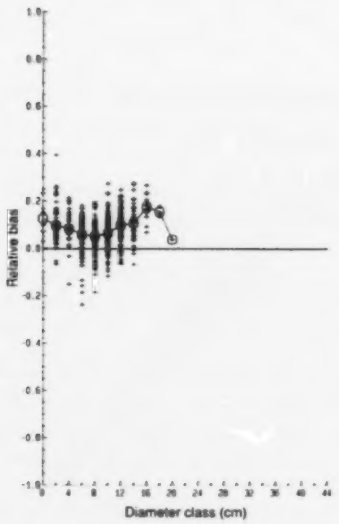

FIGURE 6. Relative bias by diameter classes of Kovats (1977) total volume equation.
Relative bias = (True volume) - (Estimated volume)/True volume.
0 = diameter-class mean relative bias.

Merchantable volume

"True" merchantable volume for each tree was calculated as total volume inside bark less a 30 cm high stump and a 10, 15, and 20 cm diameter inside bark top. Stump heights were variable among trees, and in most cases measured top (section) diameters did not coincide exactly with the 10, 15, or 20 cm top diameter limit. The desired 30-cm stump and top diameters were obtained by linear interpolation, which may have introduced some errors. The true merchantable volume factor,

$$MVF = \frac{\text{"True" merchantable volume}}{\text{"True" total volume}} = \frac{Vm}{V},$$

was calculated for each tree for each top diameter limit. "Estimated" MVF's were obtained for each tree for the 10 cm top diameter limit from Godfrey's (unpubl. memo, 1983) formulation of BCFS (1976) tabulated merchantable volume factors and Demaerschalk and Kozak's (1977) taper model. The MVF method was adopted because the BCFS (1976) estimates were available only in the ratio (merchantable volume factors) form.

The estimated and actual MVF's were plotted against dbh and the resulting graphs were visually compared. The same methods of statistical comparison as used in the total volume analysis were used to evaluate further the bias of the merchantable volume prediction equations.

Godfrey's (unpubl. memo, 1983) formulation of the BCFS (1976) factors consistently over-predicted merchantable volume; Demaerschalk and Kozak's (1977) model under-predicted merchantable volume of smaller trees and over-predicted that of larger trees (Figures 7 and 8). These observations from the graphical displays were supported by results of the t-tests (Appendix 2), which showed the bias for the 10 cm diameter limit to be statistically significant using both prediction systems. In addition, Demaerschalk and Kozak's (1977) system also tended to be biased, particularly for smaller trees, for the 15 and 20 cm top diameter limits (Figures 9 and 10).

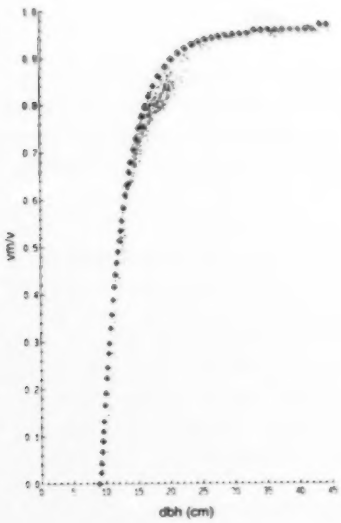

FIGURE 7. Comparison of true and Godfrey's (unpubl. memo, 1983) merchantable volume factors to a 10 cm top diameter limit.
. = True MVF
+ = Godfrey (unpubl. memo, 1983) MVF prediction.

FIGURE 8. Comparison of true and Demaerschalk and Kozak's (1977) merchantable volume factors to a 10 cm top diameter limit.
. = True MVF
+ = Demaerschalk and Kozak (1977) MVF prediction.

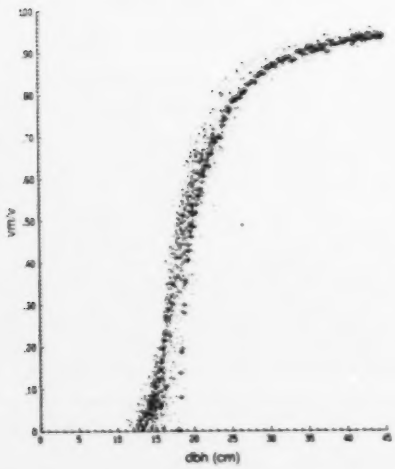

FIGURE 9. Comparison of true and Demaerschalk and Kozak's (1977) merchantable volume factors to a 15 cm top diameter limit.
. = True MVF
+ = Demaerschalk and Kozak (1977) MVF prediction.

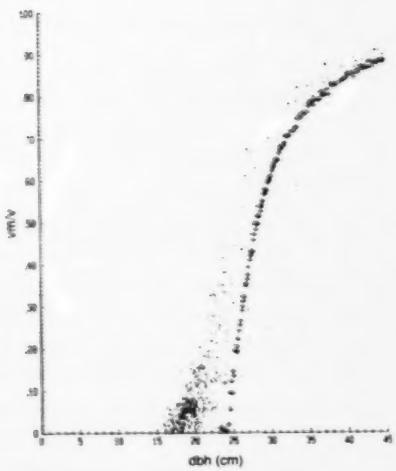

FIGURE 10. Comparison of true and Demaerschalk and Kozak's (1977) merchantable volume factors to a 20 cm top diameter limit.
. = True MVF
+ = Demaerschalk and Kozak (1977) MVF prediction.

New Volume Equations

The large data set of 2563 trees also provided an opportunity for developing new volume equations to predict total volume and merchantable volume to various top diameter limits. The new equations were not compared with the existing equations. This task, which is beyond the scope of the present paper, would require calibration of each of the four estimation methods based on the same data set prior to comparison.

Total volume

It has been hypothesized by other researchers (for example, Demaerschalk and Smith[3]) that small trees, below about 6 m in height, are different from large trees in their relationship to volume. Thus, two separate equations were fitted to the data: one for very small trees (with diameter less than 5 cm and total height less than 4 m) and the other for larger trees (with diameter equal to 5 cm or more and total height of 4 m or taller). The breaking point between the two categories was determined on the basis of Honer's (1965) arguments, which related the geometry of the tree to tree volume, as follows. The ratio of squared diameter to total volume was plotted against the ratio of breast height to total height. The two ratios defined a curvilinear relationship with positive slope until the breast height/total height ratio was about 0.32 when the slope became negative (Figure 11). The breast height/total height ratio of 0.32 corresponded to a total height of 4.0 m, the selected height breaking point. As shown by the diameter - height - volume frequency distribution of Figure 2, trees in the taller group have a minimum diameter of around 5 cm, and those in the shorter group have a maximum of 5 cm — the selected diameter breaking point.

The logarithmic model below was fitted using piecewise linear regression and dummy variables (Cunia 1973, p. 57):

$$\ln(V) = a + b^{*}\ln(dbh) + c^{*}\ln(H)$$

where

V = tree total volume inside bark in cubic metres
dbh = tree diameter at breast height (1.3 m) outside bark in centimetres
H = tree total height in metres

and a, b, and c are regression coefficients to be determined. The two regression equations were forced to be single-valued when total height equalled 4 m and dbh equalled 5 cm. The logarithmic model was chosen because it does not require a separate measure of form, and it is preferable to other model forms which demand a constant form factor (Clutter *et al.* 1983).

The new total volume equations were:

Very small trees (dbh < 5 cm or H < 4 m):

$$\ln(V) = -7.88448531 + 1.34686970 * \ln(dbh) + 0.12033677 * \ln(H) \quad (1)$$

Larger trees (dbh ≥ 5 cm and H ≥ 4 m):

$$\ln(V) = -9.896325907 + 1.83478844 * \ln(dbh) + 1.00511916 * \ln(H) \quad (2)$$

The overall regression with dummy variables from which equations (1) and (2) were derived had $R^2 = 0.99$, logarithmic standard error = 0.09514769, and logarithmic mean tree volume = -2.37234. The equations were based on 108 very small trees and 2450 large trees. Five trees were discarded as outliers because they fell outside the limits of both tree categories. Both equations have been corrected for logarithmic transformation bias (see, e.g., Miller 1984). The regression residuals showed no consistent pattern of bias (Figure 12). A height - diameter - volume prediction chart based on these equations is shown in Figure 13.

The two equations gave identical tree total volume estimate of 0.00387 m^3 for a tree with total height of 4 m and diameter of 5 cm. The BCFS (1976) equation (Table 1), for dbh equal to 44 cm and H ranging from 30 to 45 m, gave tree-volume values which were within one standard error of estimate of equation (2). Thus, cross-over from equations (1) and (2) to BCFS (1976), to estimate volumes of large trees, is relatively smooth.

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[3] Demaerschalk, J.P. and J.H.G. Smith. 1972. An examination of the problems likely to be encountered in producing volume tables for second growth. University of B.C., Faculty of Forestry, Vancouver, B.C. Unpublished report.

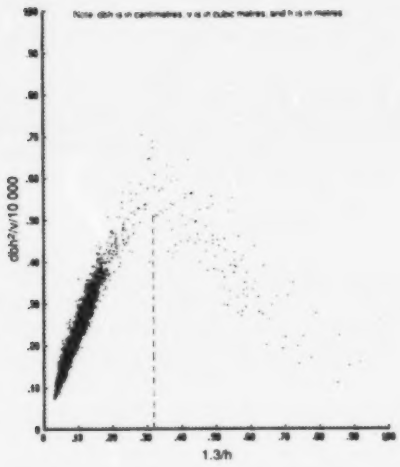

FIGURE 11. Relationship of the ratio of squared diameter breast height(dbh) to total volume (V) and the ratio of breast height (1.3) to total height (h). Diameter (dbh) is measured at breast height in centimetres, breast height and total height (h)are in metres, and total volume is in cubic metres.

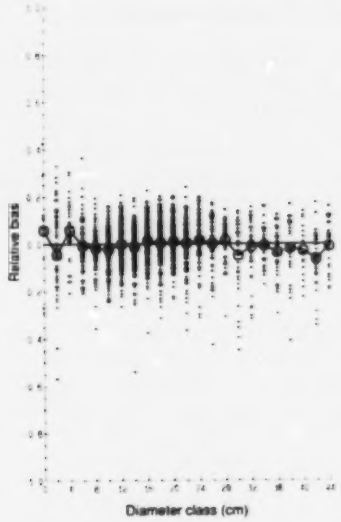

FIGURE 12. Regression residuals of the new total volume equation.

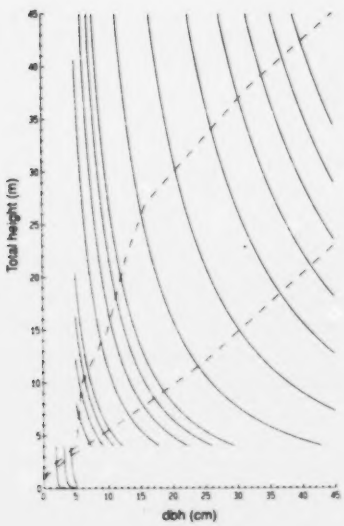

FIGURE 13. Height - diameter - volume prediction chart.
- - - = limit of original data.

Merchantable volume

Merchantable volume equations were developed separately for 10, 15, and 20 cm diameter inside bark top limits (the most common limits employed in British Columbia), using merchantable volume factors (MVF).

The MVF's when plotted over diameter at breast height described a sigmoid curve having a maximum MVF of 1 as asymptote. A logistic function (Richards 1959) was transformed and fitted by ordinary least squares, using various polynominal functions of dbh (P[X]), to the observed MVF's for each top diameter limit. The transformed logistic function was

$$\ln \frac{(MVF \quad)}{(1\text{-}MVF)} = P(x) \qquad (3)$$

The logarithmic equation (3) by its very nature goes through the origin (0,0), causing some bias. However, by transforming the ratios to $MVF' = MVF + k$ and iterating the fitting procedure for various values of $k \geq 0$, using successively finer search grids, it was forced to go through (0,t), where t = a dbh slightly less than the top diameter limit. This minimized the bias, detected from overlays (not shown) of the predicted equations placed over the original data.

The new equations of merchantable volume factors, used to estimate merchantable volume for trees under 45 cm dbh for the various top diameter limits, were:

Top diameter limit inside bark	Equation: $MVF = \frac{2}{1+e^{-P(X)}} - 1$	dbh range (cm)
10 cm	$P(X) = 5.02946535 - 48.67672151/dbh$	dbh ≥ 9.7
15 cm	$P(X) = 6.55043775 - 142.64854131/dbh + 726.71450987/dbh^2$	dbh ≥ 13.6
20 cm	$P(X) = 7.905 - 270.35199280/dbh + 2313.84955513/dbh^2$	dbh ≥ 17.1

The merchantable volume equations are depicted graphically (Figure 14), and the regression residuals plotted (Figures 15 - 17). Merchantable volume is obtained by estimating total volume from equations (1) and (2), and multiplying it by the appropriate MVF estimate.

The fitted form of equation (3) was: $\ln[\frac{MVF+1}{2-(MVF+1)}] = P(X)$

and the various statistics were:

Diameter limit	R^2	SEE	n
10 cm	.946	.236284	2207
15 cm	.949	.2305762	1614
20 cm	.940	.22843	943

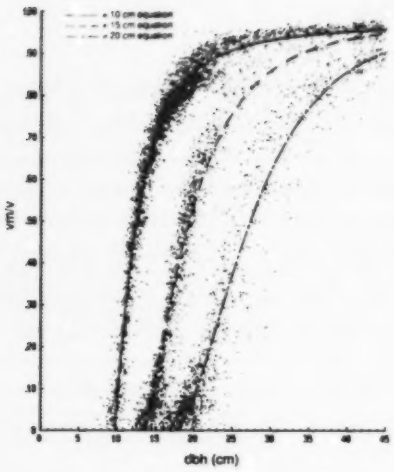

FIGURE 14. New merchantable volume factors to 10, 15, and 20 cm top diameter limits.

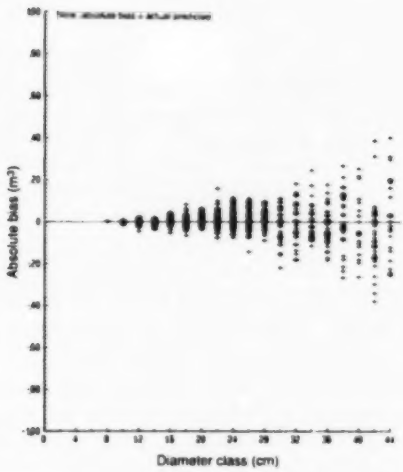

FIGURE 15. Regression residuals of the new merchantable volume factors to 10 cm top diameter limit.

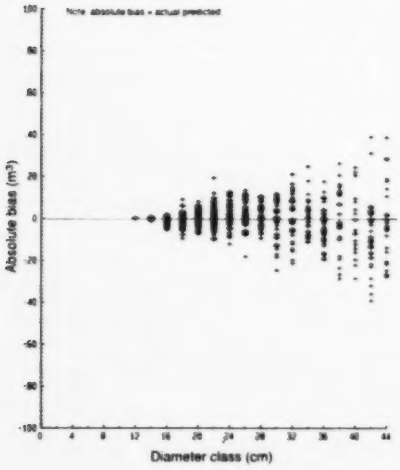

FIGURE 16. Regression residuals of the new merchantable volume factors to 15 cm top diameter limit.

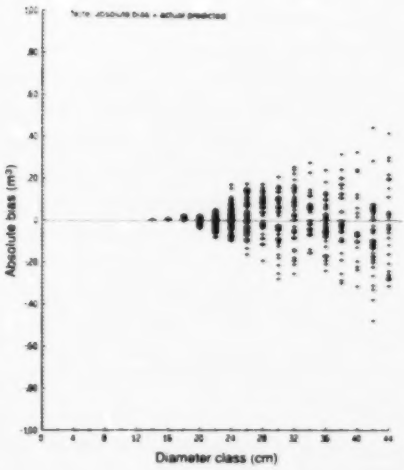

FIGURE 17. Regression residuals of the new merchantable volume factors to 20 cm top diameter limit.

DISCUSSION

Data from 2563 trees were used to evaluate the bias of existing tree volume equations of coastal Douglas-fir, although how representative these data are of stands to be inventoried or appraised now and in the near future is uncertain.

It was shown that, for the range of data in this study, the total volume prediction equations of BCFS (1976) and Godfrey (pers. comm., 1983) were generally satisfactory for trees of dbh 5 cm or more. The prediction equations of Demaerschalk and Kozak (1977), however, tended to under-predict slightly but consistently, total volume in this diameter range. The bias observed in their prediction equations might be due in part to the fact that the "true" stump volume was calculated as a cylinder, while that of the "estimate" accounted for butt swell in the stump. The bias might also be attributed to bark thickness estimating equations (Demaerschalk and Kozak 1977; equations 1 and 2 of their study). Diameter measurements were taken at the recommended heights for Kovats' (1977) volume prediction equation; however, the equation still consistently under-predicted volumes for trees under 10 m tall. Cause of the slight bias observed in Godfrey's (pers. comm., 1983) equation is uncertain. All the existing equations were, as expected, generally unsuitable for estimating the total volume of very small trees (under 5 cm dbh).

The equations previously used to predict merchantable volume have all been shown to be biased. Godfrey's (unpubl. memo, 1983) equations significantly over-predicted the merchantable volume, while Demaerschalk and Kozak's (1977) equations significantly under-predicted the merchantable volume of smaller trees and over-predicted that of larger trees. The under- and over-prediction in the latter case can probably be attributed in part to the same reasons for the total volume under-prediction. The slight over-prediction by Godfrey may be due in part to fitting an equation to tabulated (grouped) values. Merchantable volume over- or under-prediction for smaller trees may be of little consequence in applications such as appraisal, provided the proportion of small trees in the appraised population is relatively low. However, it is uncertain what the aggregate impact of the bias observed in larger trees is on inventory or appraisal applications in coastal British Columbia.

This study has provided two separate total volume prediction equations, one for very small trees and the other for larger trees. The standard error associated with the volume equations (4.01%) might be relatively large for some applications. More precise volume estimates of smaller and larger trees might be obtained by incorporating into the equations upper tree diameters (measures of form and taper; see, e.g., Hoyer 1985) and live crown ratio. Also provided are equations to predict merchantable volume factors for converting total volume to merchantable volume, for three specific top diameter limits. Generally, however, standards of what is merchantable change over time. Thus, it may be desirable to have a set of merchantable volume factors estimates that are compatible, completely general, and flexible for obtaining estimates to any specified top diameter limit. Calibration of Demaerschalk and Kozak's (1977) taper system for small trees might be useful in this case.

The new total volume and merchantable volume equations for small trees may be used in the following situations: 1) in volume summaries of thinnings in growth and yield studies, 2) in volume summaries in provenance research, and 3) in inventory and appraisal of volumes of suppressed trees or lower storey trees in multi-layered stands. In each of those cases, complete and accurate summaries — and not broad estimates — are often desirable. Downward extrapolation of equations based on mostly large trees can yield highly erroneous results or conclusions.

CONCLUSIONS

Results of the evaluation of the existing volume equations, for the range of the study data, indicated that:

1. the BCFS (1976) and Godfrey (pers. comm., 1983) total volume equations (Table 1) are generally adequate for trees with diameter 5 cm or more. On the other hand, the Demaerschalk and Kozak (1977) and Kovats (1977) equations slightly but consistently under-estimated the total volume; for smaller trees, all the total volume prediction systems were biased;
2. the Godfrey (unpubl. memo, 1983) formulation of the BCFS (1976) tables over-predicted merchantable volume for all trees, while Demaerschalk and Kozak (1977) under-predicted the volume for smaller trees and over-predicted that of larger trees;
3. it would be a useful exercise to validate the volume equations for trees of diameters more than 45 cm for coastal Douglas-fir, as well as for the other significant tree species in the province;
4. the new total and merchantable volume equations developed in this report can be adopted to estimate volumes of small coastal Douglas-fir in growth and yield studies or other applications. However, development of a system to directly measure treatment response in terms of total volume would be very useful.

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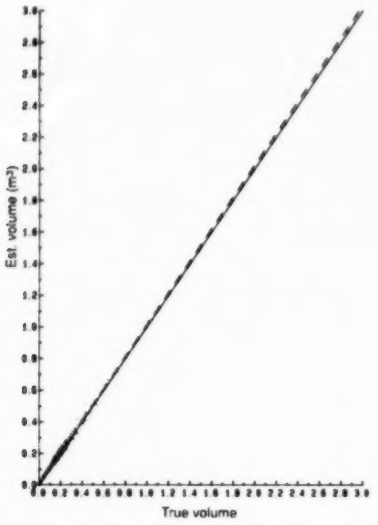
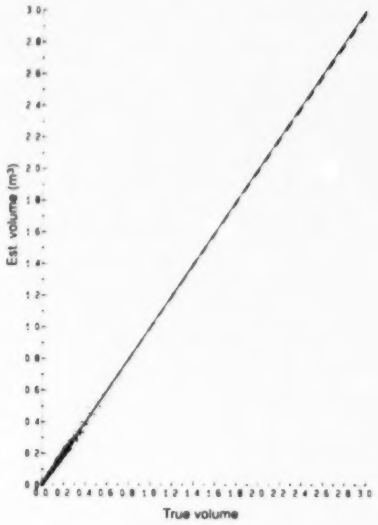
APPENDIX 1. Statistical tests of total volume estimation biases



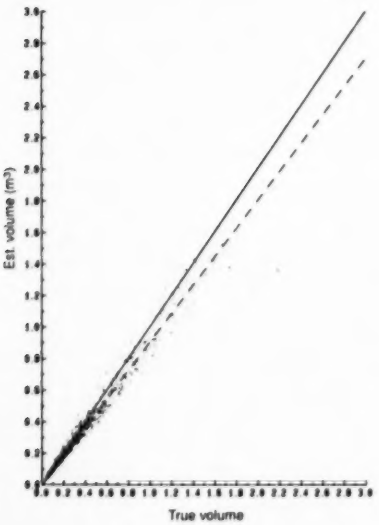
——— = Line of correspondence (y=x)

– – – = Godfrey (pers. comm., 1983) volume estimates regression on true volume (y = 0.003002424 + 1.01036604 x); the slope and intercept are not significantly different from one and zero, respectively (p =.05). (The standard errors of the slope and intercept are 0.002593575 and 0.001038035, respectively.)



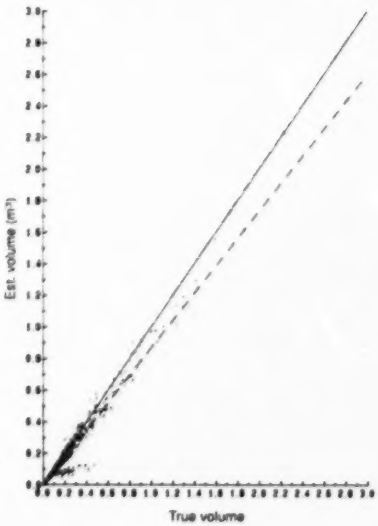
——— = Line of correspondence (y=x)

– – – = BCFS (1976) volume estimates regression on true volume (y = 0.001311742 + 0.99384163 x); the slope and intercept are not significantly different from one and zero, respectively (p =.05). (The standard errors of the slope and intercept are 0.002434926 and 0.000974539, respectively.)



——— = Line of correspondence (y=x)

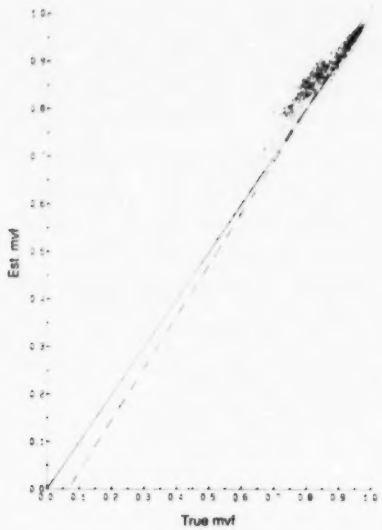
– – – = Demaerschalk and Kozak (1977) volume estimates regression on true volume (y = 0.003040352 + 0.89893433 x); the slope is significantly different from one (p =.05). (The standard errors of the slope and intercept are 0.002461953 and 0.000985356, respectively.)



——— = Line of correspondence (y=x)

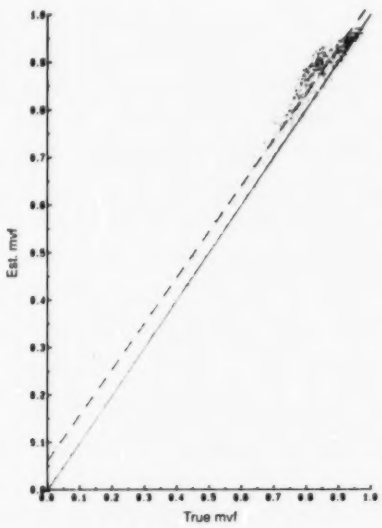
– – – = Kovats (1977) volume estimates regression on true volume (y = 0.001127558 + 0.86550345 x); the slope is significantly different from one (p =.05). (The standard errors of the slope and intercept are 0.004721195 and 0.000154784, respectively.)

APPENDIX 2. Statistical tests of merchantable volume estimation biases



——— = Line of correspondence (y=x)

– – – = Demaerschalk and Kozak (1977) MVF estimates regression on true MVF's (y = -0.07128579 + 1.08907002 x); the slope is significantly different from one (p =.05). (The standard error of the slope and intercept are 0.005554241 and 0.003827907, respectively.)



——— = Line of correspondence (y=x)

– – – = Godfrey (unpubl. memo, 1983) MVF estimates regression on true MVF's (y = 0.06092948 + 0.96572501 x); the slope is signficantly different from one (p =.05). (The standard error of the slope and intercept are 0.003360721 and 0.002316163, respectively.)